A Fixed-pointed Iteration Method of Total Variation Regularization for Image Reconstruction of ECT

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Abstract- Electrical capacitance tomography (ECT) is one of process tomography (PT) technique which developed rapidly and matured in recent years. ECT image reconstruction is a typical ill-posed problem, and its solution is unstable. Regularization is widely employed to produce proper solution. Fixed-pointed Iteration Method is proposed to solve Total Variation (TV) regularization for image reconstruction of ECT. The numerical experiments is made to show the merits and to compare with the well-known Tikhonov Regularization. The experimental results shows that the new algorithm proposed in this paper possess the advantages of high accuracy.

I. Introduction

Electrical Capacitance Tomography (ECT) is a technique for multi-phase flow measurement which developed rapidly and matured in recent years. It is used to obtain information about the distribution of a mixture of dielectric materials inside a vessel or pipe. For the advantage of being noninvasive, low in cost, simple to implement, fast in respond and high-speed image reconstruction, ECT has gained considerable momentum in recent years[1] [2].

Image reconstruction of ECT is an inverse problem. And it is ill-posed. All the image reconstruction algorithm can be divided into two kinds: regularization and non-regularization method. In order to obtain a stable solution, the regularization principle has been introduced by many authors. It is the key to improve quality and rate of image reconstruction. Fixed-pointed iteration method is proposed to solve Total Variation (TV) regularization for image reconstruction of ECT because the distribution of dielectric constant is often blocky. The experimental results indicate that the new algorithm proposed in this paper possess the advantages of high accuracy.

II. Principle of ECT

An electrical capacitance tomography system consists of three parts—the primary sensor, the data acquisition system, and an image reconstruction and display computer. The forward problem of ECT is to calculate the potential distribution for a known distribution of dielectric constant, and then to determine the capacitance measurements. The inverse problem is also called image reconstruction. The image reconstruction aims to solve the inverse problem to determine the distribution of dielectric constant within the cross-section with a limited number of measurements.

The value of the capacitance can be expressed by

$$C_{ij} = \iint_D \varepsilon(x,y) S_{ij}(x,y)\,dxdy \qquad (1)$$

$$i = 1,2,\ldots 11,\; j = i+1,\ldots 12$$

Where D is the cross-section of pipe (image area), $\varepsilon(x,y)$ is the dielectric distribution function, $S_{ij}(x,y)$ is the sensitivity distribution function between i and j.

The mathematical model of image reconstruction (1) can be written in matrix form

$$\lambda = S\mathrm{g} \qquad (2)$$

Where λ is $m\times1$ normalized measured capacitance, g is $n\times1$ grey level of the reconstructed image, S is $m\times n$ normalized sensitivity matrix, m is the number of unique electrode-pair combinations (e.g. 66 for a 12-electrode sensor), n is the number of pixels in image region. So the inverse problem is to determine g.

III. Fixed-pointed Iteration

A. Tikhonov regularization

It is well know that ECT image reconstruction is a typical ill-posed problem, and its solution is unstable. Regularization is widely employed to produce proper solution. The Tikhonov regularization is given by

$$M(\mathrm{g}) = \frac{1}{2}\left\|\mathrm{Sg} - \lambda\right\|_2^2 + \alpha\Omega(\mathrm{g}) \qquad (3)$$

Where

$\left\|\mathrm{Sg} - \lambda\right\|_2^2$ - Least Square or residual norm

$\Omega(\mathrm{g}) = \left\|\mathrm{g}\right\|_2^2$ -stabilization functional

α - Regularization parameter.

And g must be continuous. That is to say, most of the regularization methods assume the data sets to be smooth and continuous.

So the problem of solving (2) could be attributed to the following optimization problem

$$M(\mathrm{g}) = \frac{1}{2}\left\|\mathrm{Sg} - \lambda\right\|_2^2 + \alpha\Omega(\mathrm{g}) \to \min \qquad (4)$$

B. TV regularization

Since smooth solution is desirable in many applications while others require discontinuity or steep gradient to be computed. TV is a regularization technique that does takes into consideration the information that the data set is blocky and discontinuous, and in fact, distribution of dielectric constant is often blocky. So TV regularization is suitable for the problem of ECT [4]. The key feature of TV regularization is the fact that it allows for discontinuous solutions, i.e., images with sharp edges.

TV regularization is given in [3]

$$TV(g) = \frac{1}{2}\|Sg - \lambda\|^2 + \alpha TV(g) \tag{5}$$

$$TV(g) = \int_D |\nabla g| dxdy \tag{6}$$

However, $TV(g)$ is not differentiable at zero. So in order to avoid this difficulty a small positive constant value is added to (7)

$$TV(g) = \int_D \sqrt{|\nabla g|^2 + \beta^2}\, dxdy \tag{7}$$

So TV functional can be expressed as

$$T(g) = \frac{1}{2}\|Sg - \lambda\|^2 + \alpha \int_D \sqrt{|\nabla g|^2 + \beta^2}\, dxdy \tag{8}$$

This provides us with the information about the discontinuities in the image.

C. Fixed-pointed iteration method

The minimization of (8) yields a non-linear partial differential equation (PDE), which is represented as

$$(S^{\mathrm{T}} S + \alpha L(g))g = S^{\mathrm{T}} \lambda \tag{9}$$

Where

$$L(g)g = -\nabla \cdot \left(\frac{1}{\sqrt{|\nabla g|^2 + \beta^2}} \nabla g\right) \tag{10}$$

Equation (10) is non-linear. Equation (9) is also called Euler equation of (8).

The nonlinearity of (10) poses number of computation challenges. Fixed-pointed iteration is used to solve (9).We can have the fixed-pointed iterative formula

$$(S^{\mathrm{T}} S + \alpha L(g^{\mathrm{m}}))g^{\mathrm{m}} = S^{\mathrm{T}} \lambda \tag{11}$$

m is the number of iterations.(11) can be rewritten as quasi-Newton iterative formula

$$g^{m+1} = g^m - (H(g^m))^{-1} u(g^m) \tag{12}$$

Where

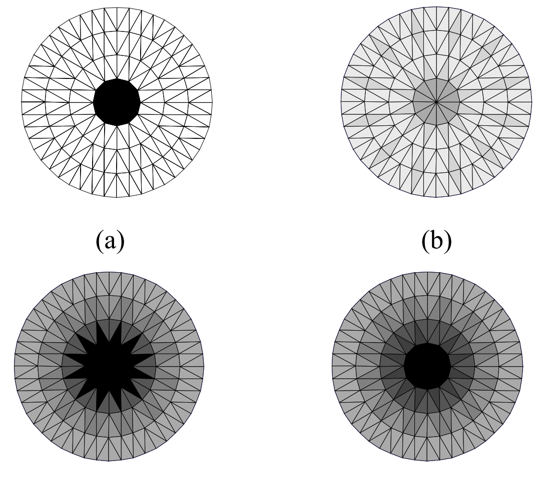
$$u(g) = S^T (Sg - \lambda) + \alpha L(g)g$$

$$H(g) = S^T S + \alpha L(g)$$

Fixed-pointed iteration is global convergence.

IV. Numerical Experiment

To evaluate the effectiveness of the method, numerical experiments have been carried out by simulating the gas/oil two-phase flow using oil (relative dielectric constant is 3) and air (dielectric constant is 1). The number of measurement electrodes is 12, and the area inside the pipe is divided into 192 elements.Fig.1 illustrates the situations tested and the images reconstructed using the new algorithm. And the images reconstructed using the LBP algorithm and Tikhonov regularization are also shown for comparison.



(a) (b) (c) (d)

Fig.1. (a) Simulated test models (b) LBP Algorithm (c) Tikhonov regularization (d) TV regularization.

The experimental results indicate that the new algorithm proposed in this paper possess the advantages of high accuracy.

V. Conclusion

Image reconstruction algorithm and technology is critical for application of ECT in practical industries. It is the key to improve quality and rate of image reconstruction. TV is a regularization technique that does takes into consideration the information that the data set is blocky and discontinuous, and it is suitable for the problem of ECT. Correctly selecting regularization parameter is crucial for the accuracy and the reconstructed image quality. At present, regularization parameter α of TV regularization is often selected from experience, and which lead to some blindness and limits of application. If the appropriate value of λ can be selected, the desired restoration can be obtained.

VI. References

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